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The importance of being light: aerodynamic forces and weight in ski jumping

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Abstract

Many contemporary world class ski jumpers are alarmingly underweight and several cases of anorexia nervosa have come to light. Athletes strive for low body weight because it gives them a major competitive advantage. In order to stop this hazardous development, changes to the regulations are being discussed, and the International Ski Federation and the International Olympic Committee wish to be proactive in safe guarding the interest of the athletes and their health.

This study of ski jumping uses field studies conducted during World Cup competitions, large-scale wind tunnel measurements with 1:1 models of ski jumpers in current equipment and highly accurate computer simulations of the flight phase that include the effects due to the athlete's position changes.

Particular attention has been directed to the design of a reference jump that mirrors current flight style and equipment regulations (2001), and to the investigation of effects associated with variation in body mass, air density, and wind gusts during the simulated flight. The detailed analysis of the physics of ski jumping described here can be used for the investigation of all initial value and parameter variations that determine the flight path of a ski jumper and will form a reliable basis for setting regulations that will make it less attractive or even disadvantageous for the athlete to be extremely light.

Keywords: Underweight; Ski jumping; Wind tunnel measurements; Aerodynamics; Computer simulation

1. Introduction

Extremely low body weight is becoming more and more prevalent in many sports such as bicycle racing, rock climbing, gymnastics, rhythmic gymnastics, figure skating, horse racing, long-distance running, and ski jumping (Smith, 1980; Müller, 2002). Athletes strive for a low body weight in these sports because it gives them a major competitive advantage. Associated with this advantage is a tendency in some of these sports to submit very young athletes to international competitions because they are lighter than older athletes of the same height (Müller and DeVaney, 1996).

This study focuses on ski jumping. Many contemporary world class ski jumpers are alarmingly underweight to the point of having a body mass index (BMI) of 16.6 $\mathrm{kg\,m^{-2}}$. According to the BMI-based WHO classification (WHO, 1995), 16.3% of the World Cup ski jumpers investigated ($n = 92$) were in thinness class I and one was in class II (Müller, 2002). Several cases of anorexia nervosa among ski jumpers have come to light during the last years. Anorexia nervosa is a serious disease causing grave mental and physical effects and even death (Becker, 1999; Sullivan, 1995; Hirschberg, 1998). A thorough analysis of the influence of weight and other variables and parameters determining the flight of a ski jumper would form the basis of new regulations that will counteract these worrying tendencies.

The earliest analytical model of ski jumping was made by Straumann, who used a 1:4 model for the first wind tunnel measurements of the lift and drag forces (Straumann, 1927). Since then many approaches to studying the physics of ski jumping have been made: Hochmuth (1958/59) introduced kinematographic techniques to study the flight path and also the biomechanical characteristics of the take-off jump; Tani and Iuchi (1971) used a 1:1 wooden model of the human body for wind tunnel measurements. In the 1980s and early 1990s Jin et al. (1995) and Ward-Smith and Clements (1982)

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performed wind tunnel measurements as well. However, most of the published wind tunnel results cannot be used for the physical analyses of today's ski jumping because the equipment and the flight style have since changed. A literature review can be found in Denoth et al. (1987) or Hubbard et al. (1989) or Schwameder and Müller (2001).

Until now, only one model of ski jumping has been presented in the literature, that takes into consideration the positional changes of the athlete during the flight phase (Müller et al., 1995, 1996), which influences the performance dramatically. This mapping of ski jumping to a computable simulation model requires precise wind tunnel data as input values to obtain highly accurate results. A representative set of data obtained with 1:1 models in a large-scale wind tunnel will be presented here.

Although low body weight is widely considered an advantage in ski jumping, there are few reliable studies of the effects of low weight on performance. The International Ski Federation (FIS) and the International Olympic Committee (IOC) recognise the health hazards associated with extremely low body weight and both organisations are interested to be proactive in safe guarding the interest of the athletes and their health.

2. Methods

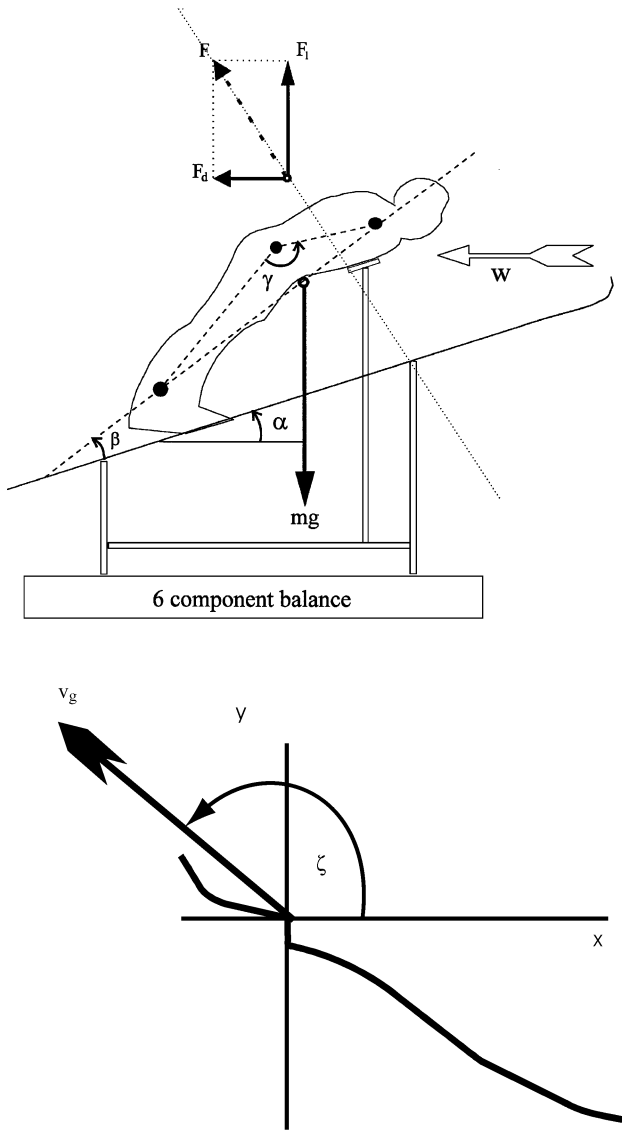
2.1. Field studies

Field studies were made during 1999/2000 World Cup in Villach (Austria; $K = 90\,\mathrm{m}$), Innsbruck (Austria; $K = 110\,\mathrm{m}$), Kulm (Austria; $K = 185\,\mathrm{m}$), and Planica (Slovenia; $K = 185\,\mathrm{m}$). The position angles of the athletes and the angle of attack of the skis (projected angles) have been measured from slides or videos taken from abeam. Typical distances between ski jumper and camera were between 20 and 30 m for the measurements from abeam. The dimensions of the object, a time resolution of 0.04 s (25 Hz Video) and a velocity of motion of approximately $25\,\mathrm{m\,s^{-1}}$, result in a deviation of the observation angle from the optical axes of $<7^\circ$. The observation angle deviation in the vertical direction corresponding to different heights of the flight paths ($\pm 1\,\mathrm{m}$) is 3°: These observation angle deviations can result in distance measurement errors of 1% and the associated errors of α, β and γ measurements were small compared to the precision limits due to the inexactness in determining the ankle, hip and shoulder reference points. A mistake of 0.02 m was to be estimated which would correspond to an angle measurement error of approximately 2.5°, or more in adverse cases. The precision limit of 2.5° (or even above) would be of relevance if it was the aim of the study to reconstruct a single flight in every detail. Here, however, mean values obtained from many measurements in the field have been used in order to define the time functions of the lift and drag areas, L and D, for a reference jump that has been used as the starting point for comparative computer simulation studies. Wind influences the performance. During major parts of the competition, the wind speed was low when compared to the wind speed limit according to the FIS regulations ($4\,\mathrm{m\,s^{-1}}$). Maximum wind speeds were (FIS official measurements): 1.6 and $2.9\,\mathrm{m\,s^{-1}}$ in Planica (first and second runs), 1.0 and $1.1\,\mathrm{m\,s^{-1}}$ in Innsbruck, 1.3 and $1.5\,\mathrm{m\,s^{-1}}$ in Villach and the wind was calm at the Kulm jumping hill. The angle nomenclature is given in Fig. 1(a).

All world-class athletes of today use the V-style. The angle V of the skis to each other was determined from digitised video images from in front. The evaluation method for the angle V has been described previously (Müller et al., 1996). More than 240 jumps of the best 30 athletes in the World Cup have been investigated and angles from more than 2400 images have been evaluated. For the design of a reference jump the best 10 athletes of each of the competitions investigated were selected and the mean values of their position angles were used in order to determine the corresponding lift and drag values in a large-scale wind tunnel.

2.2. Wind tunnel measurements

All wind tunnel measurements have been performed with 1:1 models of ski jumpers (at Arsenal Research, Vienna; Klotzsche, Dresden). Both models, A and B, with heights of $h = 1.78$ and 1.85 m, respectively, were designed with respect to the dimensions of real world class athletes in order to provide two starting points for the simulation studies. The two models are not strictly proportional in their dimensions. Ski jumping suits, skis, and other equipment components were in accordance with current FIS regulations 2000/2001. Models were used instead of real athletes in order to obtain higher positioning accuracy and precision. The tunnel has a cross-section of $5 \times 5\,\mathrm{m}^2$. A 1.8 MW motor produces a maximum wind speed of $32\,\mathrm{m\,s^{-1}}$. Air temperature was kept stable at $20^\circ\mathrm{C}$ (293 K). A circular plate (area of $0.500\,\mathrm{m}^2$) with a known drag coefficient of 1.11 (Beitz and Küttner, 1990) was used as a calibration object. The obtained blocking correction factor of 0.964 for the Arsenal Research large-scale wind tunnel has been applied for the measurements with the 1:1 models of ski jumpers. We also made comparative measurements in another wind tunnel (Dresden, Klotzsche; elliptical cross-section with 4.1 m horizontal and 3.0 m vertical diameter). For a given position of model A ($\alpha = 35.5^\circ$, $\beta = 9.5^\circ$, $\gamma = 160^\circ$, $V = 35^\circ$), the L and D values deviated by $+0.4\%$ and -1.0%, respectively, when compared to the results obtained in the Arsenal Research tunnel. The apparatus is presented



(b)

Fig. 1. (a) Schematics of the wind tunnel measurements. The figure shows the apparatus, which enabled almost all postures of athletes and skis imaginable, and demonstrates the nomenclature used for the position angles. This study used the large wind tunnel at Arsenal Research in Vienna. The tunnel has a cross-section of $5 \times 5\,\mathrm{m}^2$. A 1.8 MW motor produces a maximum wind speed of $32\,\mathrm{m\,s^{-1}}$. (b) The figure outlines a jumping hill profile and shows an advantageous angle of a gust vector v_g ($\zeta = 135°$, wind blowing up the hill; Müller, 1996). The directions of the vector v_g (in the $x-y$ plane) and the wind speed can be varied in the simulation. The gust angle ζ is positive in counterclockwise direction, i.e. $\zeta = 0$ for a wind blowing in positive x-direction, $\zeta = 90°$ for wind blowing upwards in vertical direction, etc.

schematically in Fig. 1(a). It enabled models to be positioned in all postures observed in the field. In order to minimise aerodynamic interaction, the positioning apparatus was constructed of slim steel profiles ($2 \times 3\,\mathrm{cm}^2$). The positioning apparatus was mounted on the frame of a 6 component balance (dynamometers Z6H2; $100\,\mathrm{kg} \times 9.81\,\mathrm{kg\,ms^{-2}} \mathrel{\hat{=}} 2\,\mathrm{mV\,V^{-1}}$; Hottinger-Baldwin Messtechnik, Darmstadt, Deutschland). This setup allowed the determination of all wind force components. The angles α, β, γ, and V were set before each measurement, and these values were double-checked by means of protocol photos.

According to $F_l = \rho L w^2/2$ and $F_d = \rho D w^2/2$ the values for the lift area L and the drag area D were determined, w being the wind velocity. The air density ρ was calculated according to $\rho = p/RT$, p being the actual air pressure, T the absolute temperature and R the gas constant ($288.3\,\mathrm{J\,K^{-1}\,kg^{-1}}$). The air density, according to the International Civil Aviation Organisation (Dubs, 1987), at normal atmosphere conditions is $1.225\,\mathrm{kg\,m^{-3}}$ at mean sea level, $1.15\,\mathrm{kg\,m^{-3}}$ at an elevation of 650 m, and $1.0\,\mathrm{kg\,m^{-3}}$ at 2000 m. Since jumping hills exist in elevations up to 2200 m (e.g. Park City, Olympic venue, 2002) the effect of different air densities on the aerodynamic forces must be considered.

Series of L and D measurements have been performed with values for α ranging from 0° to 40°, β from −1° to 64°, γ from 115° to 180°, and the angle V from 0° to 35°. The obtained grid of data values has been used to design reference jumps. Typical wind velocities used in the wind tunnel ranged from 24 to $25\,\mathrm{m\,s^{-1}}$. It has previously been found that a variation in the wind speed of $3\,\mathrm{m\,s^{-1}}$ does not result in measurable difference for the lift and drag coefficient (effects below 0.7%), i.e. below the accuracy of the set-up (Müller et al., 1996).

2.3. Computer-based analytical model

The universal programming environment DYNAMIX (Platzer, 1990) has been used for the computer-based modelling and for the simulation studies. The program incorporates algorithms for ordinary and partial differential equations (Hindmarsh, 1985) as well as for algebraic equations and allows adaptive, resource efficient solutions of a broad class of simulation problems. The programming environment has been designed as an interactive and portable tool (written in C). It transforms differential and algebraic equations into algorithms and data structures. Fixed time step integration (0.01 s) was sufficient to obtain a jump length calculation error smaller than 0.1 m. The simulation problem is to predict the trajectory during the flight phase and to investigate the effects of parameter and initial value variations (Koenig, 1952; Remizov, 1984; Denoth et al., 1987; Müller et al, 1995, 1996). The gravitational force and the forces due to the airstream determine the flight path of a ski jumper with a given set of initial conditions and parameters:

$$\dot{v}_x = (-F_d \cos\varphi - F_l \sin\varphi)\tfrac{1}{m},$$

$$\dot{v}_y = (-F_d \sin\varphi + F_l \cos\varphi)\tfrac{1}{m} - g,$$

$\dot{x} = v_x$,

$\dot{y} = v_y$,

where φ is the instantaneous angle of the flight path with respect to the x-axis. During the flight phase the athlete's position changes. Our modelling concept contains the dependency of F_l and F_d to the flight position by tabulated functions $L = L(t)$ and $D = D(t)$. In addition, gusts (v_g gust vector) blowing in the plane containing the flight trajectory can be selected with any desired direction and speed ($w = v_g - v$), with v being the velocity of motion along the path. Within the parameter space and the initial values of relevance for the simulation of the flight path of a ski jumper, the equations do not appear to show any stiff or complex behavior and can be solved numerically with any desired precision. The prediction accuracy predominantly depends on the accuracy of the lift and drag input values. These effects will be discussed in the result section.

3. Results

3.1. Field studies

The position angles vary during the flight phase. The least-square fits for the position angles of the best 10 athletes at each of the investigated competitions are given below, with t being the elapsed flight time. Polynomial fitting expressions have been used to maximise r^2. The flight position angles used for the design of a reference jump have been defined that way. $\alpha = 0.0843\,t^5 - 1.574\,t^4 + 11.255\,t^3 - 38.549\,t^2 + 63.724\,t - 4.7152$, $r^2 = 0.9304$; $\beta = 0.8186\,t^4 - 10.78\,t^3 + 48.927\,t^2 - 89.717\,t + 67.176$, $r^2 = 0.9246$; $\gamma = -0.0826\,t^6 + 1.6618\,t^5 - 13.26\,t^4 + 53.117\,t^3 - 110.73\,t^2 + 112.16\,t + 115.7$, $r^2 = 0.8164$. The mean values for the V angles according to the field measurements were: $V_{t=0} = 0°$, $V_{0.2} = 13°$, $V_{0.4} = 20°$, $V_{0.7} = 31°$, and $V = 35°$ for $t > 0.7$ s. The data of the V angle are based on a 2D analysis; the distance between camera and ski jumper was longer than 100 m in all cases. The mean inrun velocities (FIS official measurements) were $v_0 = 23.2\,\mathrm{m\,s^{-1}}$ in Villach, $v_0 = 24.6\,\mathrm{m\,s^{-1}}$ in Innsbruck, and $v_0 = 28.6\,\mathrm{m\,s^{-1}}$, and $v_0 = 28.5\,\mathrm{m\,s^{-1}}$ at the ski flying hills Kulm and Planica, respectively.

3.2. Wind tunnel measurements

Wind tunnel measurements obtained with model A ($h = 1.78$ m, ski length 2.60 m) and model B ($h = 1.85$ m, ski length 2.68 m) are listed in Tables 1–3. Table 1(a) (model A) and (b) (model B) shows values of lift and drag areas for typical flight positions during the first flight phase ($t = 0$–0.7 s). Tables 2(a) and (b) list lift and drag values obtained at positions of the models that represent typical positions of athletes found in the field during the main flight phase ($t > 0.7$ s). The L and D values ranged from $L = 0.736\,\mathrm{m^2}$ and $D = 0.543\,\mathrm{m^2}$ ($\alpha = 30°$, $\beta = 9.6°$, $\gamma = 160°$, and $V = 35°$) to $L = 0.849\,\mathrm{m^2}$ and $D = 0.885\,\mathrm{m^2}$ ($\alpha = 40°$, $\beta = 15.7°$, $\gamma = 160°$, and $V = 35°$). The L to D ratio maximum was found at a low angle of attack ($\alpha = 30°$), however, the higher lift values L were found at a high angle of attack ($\alpha = 40°$). Table 3 summarises the results obtained with different hip angles γ ($\gamma = 150°$, 160°, 170°, and 180°). The α-angle was fixed in this series to 35.5° and β was 5.1° or 9.5°. A selection of those values from Tables 1(a), 2(a) and 3 which were necessary to design the reference jump for model A are shown in Figs. 2(a)–(d). The graphs and the interpolating functions given in Figs. 2(a)–(d) provide all values necessary for the design of the reference jump for the 1.78 m model. The L and D data of Table 4(a) show the reference jump lift and drag areas for model A at given flight times t. Analogous values for model B ($h = 1.85$ m) are shown in Table 4(b). The angles α, β, γ, and V correspond to the mean values found in the field. Linear interpolation has been used in order to determine values between those given in Table 4(a); the graph is shown in Fig. 3(a) accordingly. The L/D ratio for the reference jump with model A is shown in Fig. 3(b).

Table 1

t (s)	α (deg)	β (deg)	γ (deg)	V (deg)	L ($\mathrm{m^2}$)	D ($\mathrm{m^2}$)	L/D
(a) *Lift and drag area values relevant for the first phase of flight ($0\,s \leqslant t \leqslant 0.7\,s$) for model A*							
0	0	63	115	0	0.285	0.397	0.72
0.2	7	49,3	135	13	0.404	0.479	0.84
0.4	14	43	145	20	0.512	0.525	0.98
0.7	25	26	155	31	0.693	0.628	1.10
(b) *Lift and drag area values relevant for the first phase of flight ($0\,s \leqslant t \leqslant 0.7\,s$) for model B*							
0	0	64	115	0	0.300	0.433	0.69
0.2	7	50	135	13	0.445	0.510	0.87
0.4	14	43	145	20	0.550	0.516	1.07
0.7	25	26	155	31	0.696	0.604	1.15

Table 2

α (deg)	β (deg)	γ (deg)	V (deg)	L (m^2)	D (m^2)	L/D
(a) *Lift and drag area values relevant for flight times longer than 0.7 s for model A*						
30	9.6	160	35	0.736	0.543	1.36
30	14.4	160	35	0.758	0.613	1.24
30	20.2	160	35	0.764	0.700	1.09
35.5	0.5	160	35	0.756	0.584	1.29
35.5	5.1	160	35	0.784	0.644	1.22
35.5	9.5	160	35	0.811	0.693	1.17
35.5	15.2	160	35	0.815	0.769	1.06
40	0.4	160	35	0.806	0.683	1.18
40	5.0	160	35	0.832	0.759	1.10
40	9.4	160	35	0.840	0.826	1.02
40	15.7	160	35	0.849	0.885	0.96
(b) *Lift and drag area values relevant for flight times longer than 0.7 s for model B*						
30	8.0	160	35	0.738	0.546	1.35
30	13.2	160	35	0.809	0.626	1.29
30	19.1	160	35	0.831	0.708	1.17
35.5	1.7	160	35	0.752	0.587	1.28
35.5	5.6	160	35	0.785	0.634	1.24
35.5	9.8	160	35	0.812	0.698	1.16
35.5	13.9	160	35	0.818	0.741	1.10
40	−1.0	160	35	0.812	0.704	1.15
40	4.5	160	35	0.834	0.788	1.06
40	8.5	160	35	0.846	0.838	1.01

Table 3
Lift and drag values obtained at various hip angles γ

α (deg)	β (deg)	γ (deg)	V (deg)	L (m^2)	D (m^2)	L/D
35.5	5.1	180	35	0.772	0.696	1.11
35.5	9.5	180	35	0.770	0.724	1.06
35.5	5.1	170	35	0.791	0.654	1.21
35.5	9.5	170	35	0.808	0.699	1.16
35.5	5.1	160	35	0.784	0.644	1.22
35.5	9.5	160	35	0.811	0.693	1.17
35.5	5.1	150	35	0.773	0.641	1.21
35.5	9.5	150	35	0.799	0.686	1.16

3.3. Computer simulation studies

Reference jumps were established according to position angles measured in the field using the sets of wind tunnel measurements for the models. The L and D values have been kept constant from $t = 5.5\,\mathrm{s}$ on since it is not useful to consider a landing phase from this time on, when the flight time varies due to different simulation protocols.

3.3.1. Reference jump results

The nonlinear and coupled equation of motion precisely describe the trajectory of any flying object if the initial conditions and the lift and drag areas, that are changing with changing positions, are known precisely. The inrun velocity is being measured officially by the FIS during the competitions and velocity values are displayed with a $0.1\,\mathrm{km\,h^{-1}}$ resolution. Using the L and D values of Fig. 3(a) (reference jump for model A) for the computer simulation of the flight resulted in the plots shown in Figs 4(a)–(c). For this simulation series the profile of the jumping hill in Sapporo ($K = 120\,\mathrm{m}$) has been used. The profile and the flight trajectory are shown in 4(a). The jump length 1 was 112.5 m when using 65 kg for the mass of the athlete with equipment, an approach velocity $v_0 = 26.0\,\mathrm{m\,s^{-1}}$, and a take off velocity perpendicular to the ramp $v_{p0} = 2.5\,\mathrm{m\,s^{-1}}$ (due to the athlete's take-off jump). The flight time t_l was

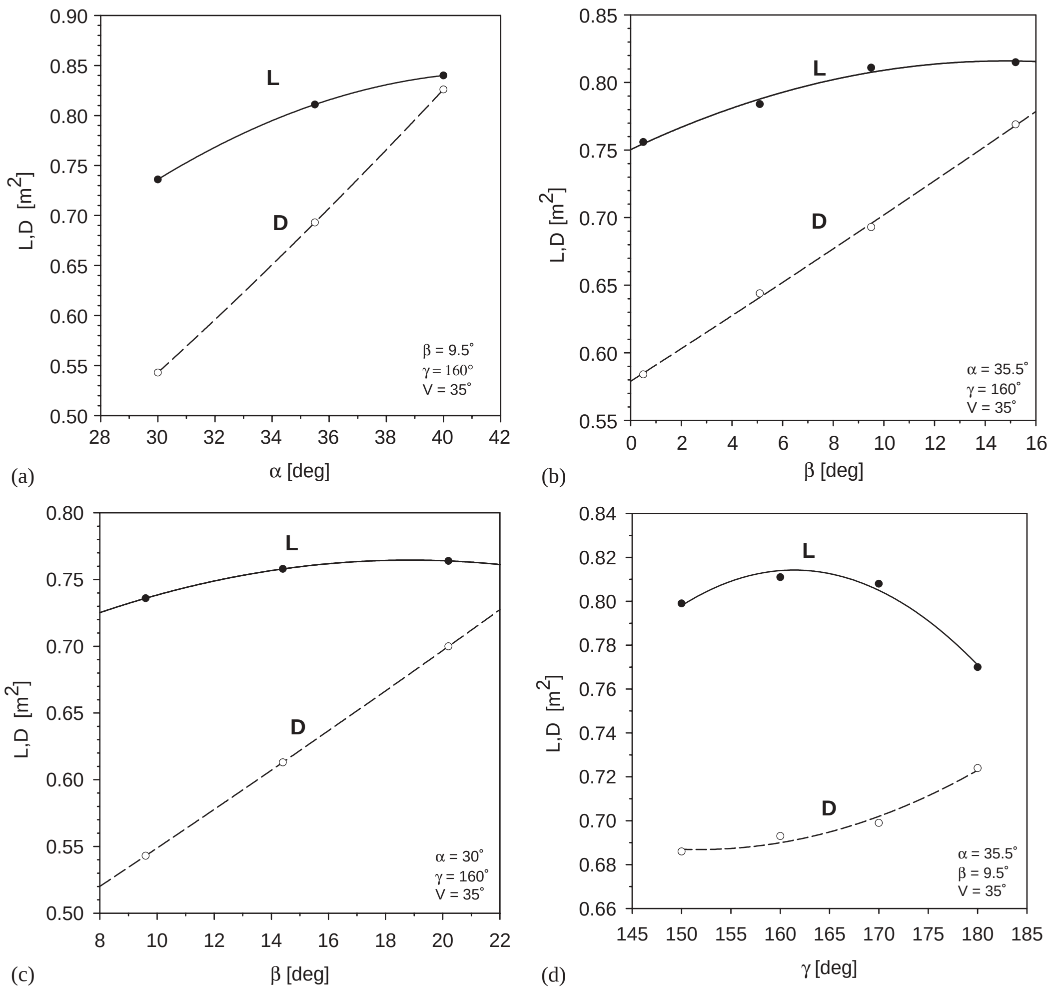


Fig. 2. (a)–(d) show the L and D values for model A (height $h = 1.78\,\mathrm{m}$). (a) Different angles of attack α. The body position was held constant ($\beta = 9.5°$ and $\gamma = 160°$) and the angles of attack were 30°, 35.5° and 40°. The opening of the skis was held constant at $V = 35°$. The interpolating functions are: $L = -0.43903 + 0.060743\alpha - 7.192 \times 10^{-4}\alpha^2$; $D = -0.032061 + 0.01232\alpha + 2.283{\times}10^{-4}\alpha^2$. (b) L and D values depending on the body to ski angle β. The values shown have been taken at $\alpha = 35.5°$, $\gamma = 160°$, and $V = 35°$. The interpolating functions are: $L = -0.645718 + 0.0126185\beta - 3.348 \times 10^{-4}\beta^2$; $D = 0.408434 + 0.01364\beta + 3.9308 \times 10^{-5}\beta^2$. (c) L and D values depending on the body to ski angle β. The values shown here have been taken at $\alpha = 30°$, $\gamma = 160°$ and $V = 35°$. The interpolating functions are: $L = 0.75037 + 8.86746{\times}10^{-3}\beta - 2.99665 \times 10^{-4}\beta^2$; $D = 0.578995 + 0.01201\beta + 2.91724{\times}10^{-5}\beta^2$. (d) L and D values depending on the hip angle γ. The body to ski angle β and the angle of attack α were held constant ($\beta = 9.5°$ and $\alpha = 35.5°$). The opening of the skis was held constant at $V = 35°$. The interpolating functions are: $L = -2.442 + 0.04035\gamma - 1.25 \times 10^{-4}\gamma^2$; $D = 1.722 - 0.01365\gamma + 4.5 \times 10^{-5}\gamma^2$.

4.32 s. Fig. 4(b) displays the velocity of motion $v = v(x)$ and the horizontal component of v: $v_x = v_x(x)$. The velocity of motion v shows a minimum at $x = 28.65\,\mathrm{m}$ (corresponding to a flight time $t = 1.16\,\mathrm{s}$), whereas its horizontal component decreases monotonously. The landing velocity v_l was $28.69\,\mathrm{m\,s^{-1}}$ and the component perpendicular to the hill (at landing) v_{pl} was $2.93\,\mathrm{m\,s^{-1}}$, which corresponds to the landing velocity of a jump onto a horizontal plane from a height $h_l = 0.44\,\mathrm{m}$ (h_l equivalent landing height). The solid line in Fig. 4(c) shows the lift force F_l and the broken line shows the drag force F_d. The L and D values obtained with model B resulted in a simulated jump length of 116.7 m when using an unchanged value of 65 kg for the mass. A mass increase to 68.3 kg for the taller model B leads to the same jump length as obtained with model A ($l = 112.5\,\mathrm{m}$). The approach velocity was the same for all three simulations.

3.3.2. *Estimation of the accuracy of the simulation*

The simulation output accuracy depends primarily on the accuracy of $L(t)$ and $D(t)$, which is determined by:

1. The inexactness of the determination of the position angles. We assume, for example, a measurement error for α of $+2°$. A simulation using the associated values for L and D for the main flight phase (from $t = 1.0\,\mathrm{s}$ on) resulted in a 3.5 m reduction in jump length (109 m compared to 112.5 using the Sapporo $K = 120\,\mathrm{m}$ jumping hill profile). Analogously, an

Table 4

t (s)	0	0.2	0.4	0.7	1.0	1.2	1.5	2.0	4.0	5.5
(a) *Lift and drag values for the reference jumps for model A*										
L (m^2)	0.275	0.389	0.494	0.668	0.738	0.766	0.774	0.786	0.795	0.784
D (m^2)	0.383	0.462	0.506	0.605	0.626	0.644	0.662	0.697	0.732	0.686
L/D	0.72	0.84	0.98	1.10	1.18	1.19	1.17	1.13	1.09	1.14
α (deg)	0	7	14	25	30.2	32.6	34.8	36.1	37.1	36.2
β (deg)	63	49	43	26	16.4	13	10.4	10.3	10.8	9.3
γ (deg)	115	135	145	155	159	159	159	158	161	164
V (deg)	0	13	20	31	35	35	35	35	35	35
(b) Lift and drag values for the reference jumps for model B										
L (m^2)	0.289	0.429	0.530	0.671	0.801	0.808	0.775	0.788	0.798	0.785
D (m^2)	0.417	0.492	0.497	0.582	0.652	0.654	0.656	0.692	0.731	0.683
L/D	0.69	0.87	1.07	1.15	1.23	1.24	1.18	1.14	1.09	1.15
α (deg)	0	7	14	25	30.2	32.6	34.8	36.1	37.1	36.2
β (deg)	64	50	43	26	16.4	13	10.4	10.3	10.8	9.3
γ (deg)	115	135	145	155	159	159	159	158	161	164
V (deg)	0	13	20	31	35	35	35	35	35	35

The angles α, β, γ, and V correspond to the mean flight positions found in the field.

increased value of β by $+2°$ resulted in a 2.3 m reduction in jump length in the simulation.

2. The inaccuracy of the lift and drag measurements in the wind tunnel due to the inexactness of the model position (angular error $<1°$) and the insufficiency of blocking corrections when objects of different geometry are to be investigated. An increase in both L and D by $+2\%$, compared to the reference jump values, increases the simulated jump length by 1.6 m, whereas an increase of D alone by 2% would reduce it from 112.5 m to 109.6 m.

3.3.3. *Reliability of the comparative simulation studies*

The very high reliability of the comparative simulation studies can be illustrated by assuming a large error in the input L and D values, which are the limiting parameters for the accuracy of the simulation. If, for instance, the effects of different masses are to be investigated, we obtain the following results (Sapporo $K = 120$ profile, reference jump (A): A reduction of the mass from 65 to 63 kg results in a jump length increase from 112.5 to 115.0 m (+2.5 m). When using the same protocol, but with L and D values increased by 3% (assumption of a large L and D measurement error), we find a jump length of 115.0 m with mass being 65 kg and 117.7 m with mass set to 63 kg. The predicted jump length increase of 2.7 m due to the 2 kg reduction in mass is, in this case, essentially the same as above. A simulation protocol with only L set higher by 3% throughout the flight resulted in +2.7 m jump length increase due to 2 kg less mass, and finally D set 3% higher resulted in a +2 m jump length increase due to the 2 kg mass reduction. The prediction error for the worst of these cases was less than 0.3 m (for comparison: the length measurement resolution in a real competition is 0.5 m).

3.3.4. *Consideration of the elevation (air density)*

For this simulation series, the Olympic jumping hill profile in Park City (Olympic venue 2002) has been used ($K = 120$ m). According to the FIS jumping hill certificate we used an inrun-velocity of 25.8 m s^{-1}. All other parameters were set as has been described above for the Sapporo ramp ($v_{p0} = 2.5\,\mathrm{m\,s^{-1}}$; $m = 65$ kg). Using an air density of $\rho = 1.15\,\mathrm{kg\,m^{-3}}$ (corresponding to an elevation of 650 m) the resulting jump length was 115.9 m. With $\rho = 1.0\,\mathrm{kg\,m^{-3}}$, which is realistic for Park City (elevation above 2000 m), a jump length of only 106.3 m resulted. The difference in the air density used for these simulations could be compensated by an increased inrun-velocity of $v_0 = 26.27\,\mathrm{m\,s^{-1}}$, however, this considerably effects the height above ground.

3.3.5. *Influence of body mass on the performance*

We used the profile of the jumping hill in Park City ($K = 120$ m) and started out from a perpendicular velocity v_{p0} (due to the athlete's jumping force) of 2.5 m s^{-1} (Virmavirta et al., 2001), which resulted in a jump length of $l = 115.9$ m. An approach velocity of $v_0 = 26.27\,\mathrm{m\,s^{-1}}$, a mass $m = 65$ kg (athlete with equipment) and an air density of $\rho = 1.0\,\mathrm{kg\,m^{-3}}$ have been used. The corresponding trajectory $y = y(x)$, as well as the trajectories for $m = 55$ and 75 kg, and the profile of this jumping hill are shown in the upper panel of Fig. 5(a). The mass m profoundly influences the jump length l and the velocity of motion along the flight path $v = v(x)$. The jump length l was only 106.2 m at $m = 75$ kg and increased to 125.7 m at $m = 55$ kg. As can be seen in the lower panel of Fig 5(a), a light athlete

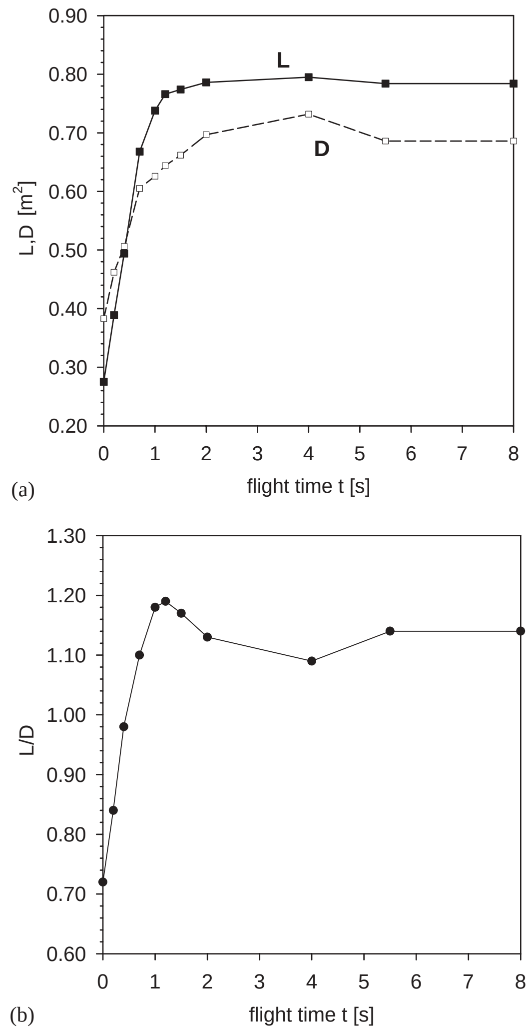


Fig. 3. (a) The figure shows the L and D values of the reference jump for model A. The values are functions of time reflecting the athlete's position changes during the flight. (b) L/D ratio for the reference jump with model A.

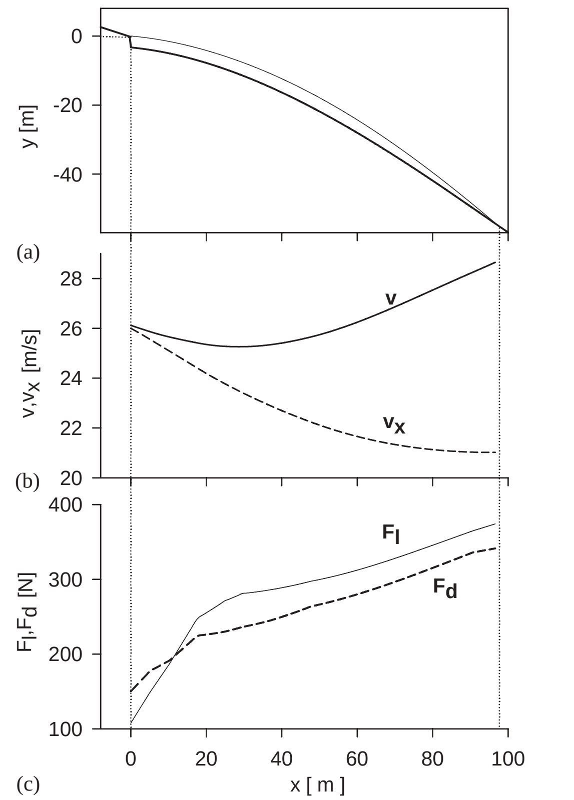


Fig. 4. Results with the reference jump for model A. Fig. 4(a) shows the profile of the jumping hill in Sapporo and the trajectory $y = y(x)$. Jumping hill parameters for Sapporo ($K = 120$ m): $a = 11^\circ$, $b = -37^\circ$, $c = 35^\circ$, $H(K) = 59.449$ m, $N(K) = 103.391$ m, $R_1 = 105$ m, $R_2 = 120$ m, $M = 20$ m, $T = 7$ m, $S = 3.3$ m. The velocity of motion v (solid line) and the horizontal component of this velocity v_x (broken line) are shown in Fig. 4(b). Fig. 4(c) shows the lift force F_l and drag force F_d acting on the athlete and his equipment. The air density was set to $1.15\,\mathrm{kg\,m^{-3}}$; the mass of the athlete with equipment was 65 kg.

moves with a noticeably lower velocity. Fig. 5(b) shows the according heights above ground h_g for $m = 55$, 65, and 75 kg.

When an athlete with a given height loses weight, his frontal surface area will decrease. The use of a parallelepiped model instead of the athlete ($V = 1.20\,\mathrm{m} \times 0.30\,\mathrm{m} \times 0.1806 = 0.065\,\mathrm{m}^3$; $\rho = 1000\,\mathrm{kg\,m}^{-3}$, -3, thus $m = 65$ kg) results in a 1.56% decreased area when the mass is decreased from 65 to 63 kg. Approximately half of the aerodynamic forces acting on the system (jumper plus skis) are contributed by the skis and the other half by the athlete. Thus decreased L and D values by $0.5 \times 1.56\% = 0.78\%$ in the simulation give an estimation of the effect of decreased area due to a shrinkage of the body as a result of weight loss (with the height kept constant). The jump length of 112.5 m (reference jump A; Sapporo $K = 120$ m) increases when m is set to 63 kg (instead of 65 kg) to 115.0 m. The additional consideration of a 0.78% L and D value decrease results in a jump length of 114.4 m.

3.3.6. Consideration of wind

Jumps to or beyond the K-point of a hill can usually be performed in reality only with the help of wind. For Park City ($K = 120$ m), the effect of a modest gust ($v_g = 3\,\mathrm{m\,s^{-1}}$) blowing during the whole flight from an advantageous angle ($\zeta = 135^\circ$, compare with Fig. 1(b) on the flight trajectory and on the velocity of motion is shown in Fig. 5(c). This wind condition resulted in increased jump lengths of $l = 120.0$ m (compared to 106.2 m, $m = 75$ kg), $l = 128.7$ m (compared to 115.9 m, $m = 65$ kg), and $l = 136.8$ m (compared to 125.7 m; $m = 55$ kg).

Using the same wind vector v_g in the simulation, a pronounced increase in jump length l occurred for all sizes of jumping hills: The jump length 1 was 128.9 m when using the Sapporo hill profile ($K = 120$ m), 178.9 m for the Kulm ($K = 185$ m), and 99.5 m for

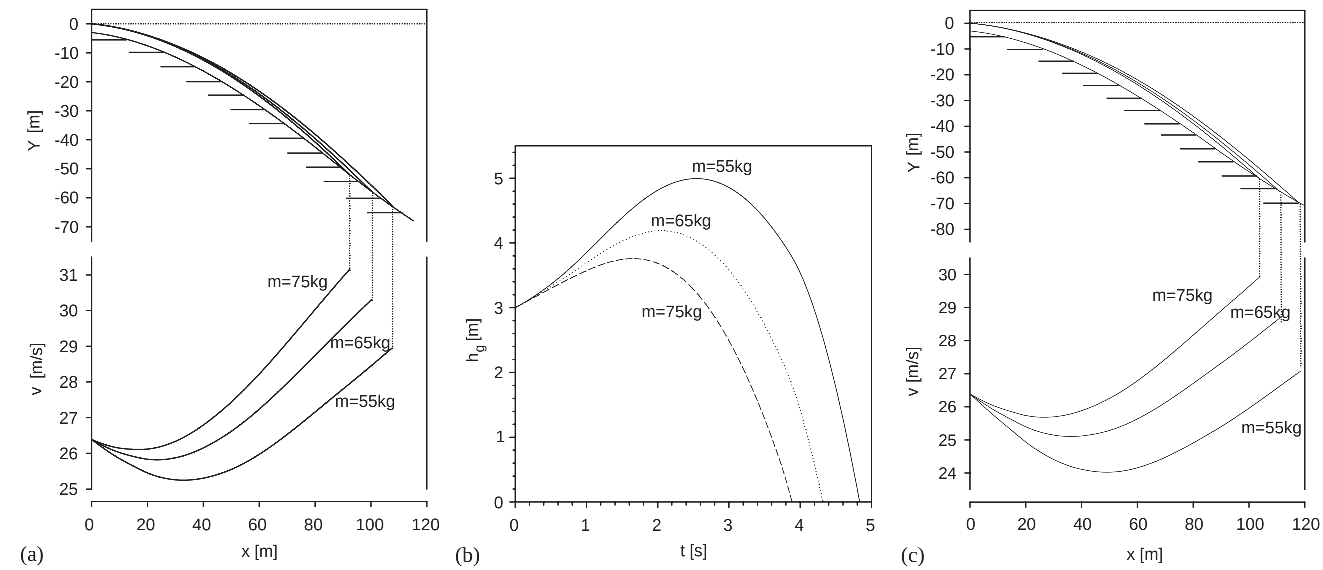


Fig. 5. (a) Simulated jumps using the L and D tables from the reference jump for model A and the profile of the jumping hill in Park City. Jumping hill parameters for Park City ($K = 120\,\mathrm{m}$): $\alpha = -10.5°$, $\beta = 35°$, $\beta_P = 38°$, $\beta_L = 37.77°$, $\gamma = 35°$, $h = 59.52\,\mathrm{m}$, $n = 103.51\,\mathrm{m}$, $r_1 = 93\,\mathrm{m}$, $r_2 = 105\,\mathrm{m}$, $r_L = 356.5\,\mathrm{m}$, $l_1 = 18.67\,\mathrm{m}$, $l_2 = 13.90\,\mathrm{m}$, $t = 6.7\,\mathrm{m}$, $s = 3\,\mathrm{m}$. For all jumping hills approved by the FIS the parameters can be found in the FIS Certificates of Jumping Hills. The trajectories and velocities for three different masses (55, 65 and 75 kg) are shown. The approach velocity v_0 was $26.27\,\mathrm{m\,s^{-1}}$, the air density $\rho = 1.0\,\mathrm{kg\,m^{-3}}$ and v_{p0} was $2.5\,\mathrm{m\,s^{-1}}$. The gust velocity v_g was set to 0. (b) The figure shows the height above ground h_g for three different masses as a function of the flight time t. The solid line shows a jump simulation using a mass of $m = 55\,\mathrm{kg}$, the dotted line for $m = 65\,\mathrm{kg}$, and the broken line for $m = 75\,\mathrm{kg}$. (c) This figure is analogous to 5(a), however, in this case a gust is blowing constantly with $v_g = 3\,\mathrm{m\,s^{-1}}$ from an advantageous direction ($\zeta = 135°$) during the whole flight was used.

Villach ($K = 90\,\mathrm{m}$), whereas the jump lengths obtained in the according simulations without wind were only 112.5, 158.5 and 89.5 m, respectively. Additionally, the landing velocities v_l decreased (Sapporo: v_l without wind was 28.7 and $27.0\,\mathrm{m\,s^{-1}}$ with wind; Kulm 30.3 and $28.4\,\mathrm{m\,s^{-1}}$; Villach 27.7 and $26.1\,\mathrm{m\,s^{-1}}$).

4. Discussion

Since 1993 the equipment regulations, particularly for ski length, the jumping suit, and front ski length have changed considerably. This has had a major impact on the flight style and on the aerodynamic forces acting on the athlete. The body to ski angle β has increased, because the FIS had followed our suggestion to limit the percentage of front ski to total ski length to 57% (Müller et al., 1995) in order to ease the pitching moment balance and thus to reduce the tumbling risk. The flight velocity has increased due to thinner jumping suits that are now being used.

Particular attention has been directed to the design of reference jumps that mirror the current (2001) flight style and equipment regulations. The reference jumps defined here are based on field studies of flight positions of current ski jumping and on the corresponding measurements in a large-scale wind tunnel. The use of models increased the positioning accuracy when compared to measurements with athletes. It is important to point out, that the ratio L/D had a maximum at a low angle of attack $\alpha = 30°$, while the maximum L value was found at a high angle of attack $\alpha = 40°$. It is not only the ratio L/D that determines the performance during the flight, it also is the absolute value of L, which should be high, and the absolute value of D, which should be low, that influence the performance effectively. The difficult optimisation problem associated with this has to be solved by the athlete in real time. A simplified optimisation approach to this problem has been published by Remizov (1984).

The accuracy of this analytic model is essentially determined by the accuracy of the input lift and drag values $L_{(t)}$ and $D_{(t)}$. A large assumed error of 2% in both L and D would result in a jump length prediction error of 1.6 m, an error of 2% in D alone in a prediction error of 2.9 m (Sapporo jumping hill profile, $K = 120\,\mathrm{m}$, reference jump A). The assumption of even higher errors for the input values L and D does not noticeably effect the reliability of comparative simulation studies. For example, the simulation protocol for reference jump A resulted in $l = 112.5\,\mathrm{m}$ (Sapporo, $K = 120\,\mathrm{m}$). A change of the mass in the protocol from 65 to 63 kg resulted in $l = 115.0\,\mathrm{m}$. The assumption of an error of +3% for the input L and D values resulted in $l = 115.0\,\mathrm{m}$ with the mass being 65 kg and $l = 117.7\,\mathrm{m}$ using a mass of 63 kg. The two predictions concerning the effect of reduced mass do not noticeably differ (the measurement accuracy in a real competition is 0.5 m). In the real world, the change of one parameter might influence others. In the computer simulation, several parameters or initial conditions can be changed simultaneously as well. For instance, a simulation protocol that, in addition to the reduction of mass from 65 to 63 kg, considers the decrease of frontal surface area associated with a loss of weight by using appropriately decreased values for L and D results in $l = 114.4$ m, i.e. a jump length increase of 1.9 m. In the real world, an additional increase is likely because lower weight effects the torque balance and would allow an aerodynamically advantageous flight position. This additional effect has not been considered here.

The reference jumps introduced here and the computer model of ski jumping enable the quantification of the effects associated with all parameters and initial values that determine the flight path of a ski jumper. Beside the impact of low mass, we also studied the influence of air density, which decreases with increasing elevation. The aerodynamic forces are proportional to the air density. Using an air density of only $1.0\,\mathrm{kg\,m^{-3}}$ in the simulation, which is appropriate for the high elevation of Park City, resulted in too short jumps. The design of the jumping hill profile should take into consideration effects associated with the low air density. Computer simulation to optimise a jumping hill profile can avoid unnecessary costs associated with subsequent hill modifications (Müller, 1997).

The computer simulation shows that the jump length markedly increases with decreasing weight. Anthropometrical studies indicate that some of the athletes and their coaches go beyond the limits of reason (Müller, 2002). Regulations that recompense heavier athlete's ballistic disadvantages are urgently needed. Several possibilities such as ski length or jumping suit regulations, that also consider body weight will be discussed in the near future and the data presented here will form a reliable basis for the improvement of regulations. Another possible approach would be to have different inrun starting points depending on relative body weight. While many suggestions may be valid from the standpoint of physics, their suitability for use in the field must be thoroughly assessed prior to proposing any changes to current regulations.

Other than weight, wind also dramatically affects performance. In both cases, with low body weight or with advantageous wind, the athlete has not only the advantage of flying further but also the touch down is eased due to a lower velocity. During the last decade the development of the equipment and flight style has resulted in a selection of light and extremely light athletes. It is up to us to guide future developments in a direction that is justifiable in terms of fairness and the health of the athlete.

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