USING INTERACTIVE SIMULATIONS TO ENHANCE CONCEPTUAL DEVELOPMENT AND PROBLEM SOLVING SKILLS

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Research during recent decades indicates that traditional didactic instruction is not producing the student learning that we desire. More importantly, the research is helping in the development of new pedagogical strategies and curriculum that are improving student achievement. We describe one effort based on this research. The goals are to help students: develop qualitative representations and imagery so that they can reason effectively without math about physical processes; learn to use the symbolic language of physics by linking it to other representations such as words, sketches, diagrams, and graphs; develop the skills needed to solve complex multipart problems; learn to learn; and develop the skills needed to work effectively in groups. Interactive multimedia plays an important role in this learning system.

I. INTRODUCTION

Frederick Reif suggested that the education system is analogous to a quantum mechanical operator:

$$\psi = \varepsilon \ \psi_o . \qquad (1)$$

An educational transformation operator ε helps students move from an initial intellectual-performance state ψ_o to a desired final state ψ (1). Such a system requires that:

- the initial intellectual-performance state ψ_o be characterized;
- the desired final intellectual-performance state ψ be characterized; and
- an transformation operator ε —a physics-learning system—is devised and implemented that causes the desired transition.

The Initial State ψ_o

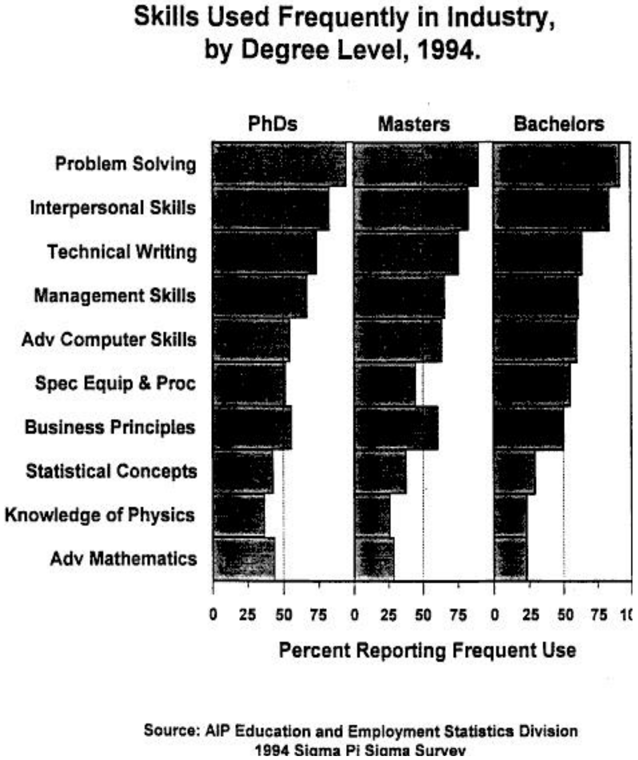
Research about learning has led to a much better understanding of students' initial states. Students have considerable pre-knowledge that often differs from accepted physics beliefs. Students have attitudes about learning and about the nature of science that make learning difficult. In addition, cognitive research provides important information concerning the way a person’s sensory system and mind interacts with the world and processes information. An educational transformer that ignores this initial state has little chance of causing a transition.

The Final State ψ

For the final state, we would like the student to have:

- a good understanding of the fundamental concepts of physics that allows them to reason effectively about physical processes without using math,
- the ability to quantitatively apply the concepts of physics to analyze and solve complex physical problems such as are encountered in the real world,
- general analytical skills that are useful for life in general and for science in particular,
- an organized knowledge structure that can be accessed efficiently, and
- skills that match the needs of the twenty-first century workplace.

A US Department of Labor SCANS Report (2) indicated that businesses and industrial firms want future employees who: have learned how to learn; possess listening and oral communication skills; are adaptable because of creative-thinking and problem-solving expertise; and are effective in groups using interpersonal skills, negotiation skills, and teamwork. A recent survey (3) by the American Institute of Physics (AIP) asked former physics majors who are now in the workplace to identify the most important skills needed for their work (see Figure 1). Solving complex poorly-defined problems was rated the most important skill followed closely by the interpersonal skills needed to work effectively in groups and by technical writing.



Skills Used Frequently in Industry, by Degree Level, 1994.

Source: AIP Education and Employment Statistics Division
1994 Sigma Pi Sigma Survey

FIGURE 1. Important Workplace Skills for Physicists. (AIP Education and Employment Statistics Division, 1994 Sigma Pi Sigma Survey).

Is Traditional Education Causing the Desired Transition?

Twenty plus years of research about learning in physics indicates that traditional instruction fails to shift students very far from their initial states. Students leave courses with little qualitative understanding (4-21). They use plug-and-chug equation-centered problem-solving techniques (22-31). Their minds hold a plethora of facts and equations that have little meaning or organization (32-37). Even with our sincere efforts to make the traditional system succeed, it seems more like an identity operator *I* than a transformation operator:

$$\psi_o = I \ \psi_o \qquad (2)$$

The final state differs little from the initial state. How can we do better?

The research about learning has led to new curricula and to new pedagogical strategies. Student learning has improved in response to this innovation. In addition, many of the pedagogical strategies that are used prepare students better for the twenty-first century workplace. This paper describes one such effort.

II. AN ALTERNATIVE TRANSFORMATION OPERATOR

The limited time interval given to us for introductory physics instruction means that we must make choices when selecting student learning goals—we cannot do everything. The AIP Survey of former physics students and the US Department of Labor SCANS Report support, I believe, a less is more philosophy. Less is more received almost unanimous support by the recent AIP Introductory University Physics Project effort to reform the introductory university physics course. With these reports and projects in mind, we choose the following goals and pedagogical strategies for our introductory course.

First, physics knowledge was judged low in importance by the former physics majors in the AIP Survey. On the other hand, the SCANS report indicated that learning to learn is very important. It seems likely that a scientist or engineer, when confronted with a new problem, uses special knowledge that is not learned in school. Yet there must be some general background knowledge that helps in acquiring this special knowledge. In physics, we could select a reduced content of the most important principles introduced in a format in which students take more responsibility for acquiring that knowledge. The student would gain a general background of knowledge and would start to develop the skills needed to learn to learn.

Second, the development of problem-solving skills was judged very important. The problems in the real world are not the well-defined end-of-chapter problems that we find in textbooks. They are instead poorly-defined problems. The concepts, the equations, and the values of quantities needed to solve these problems are not provided. If we reduce the content, we can spend more time helping students develop the skills needed to address more complex problems.

Third, according to the AIP Survey, 80 percent of our former physics majors either work in a group or supervise a group; helping students learn to work effectively in groups is one of the most important workplace skills. Fortunately, this is a win-win situation. Student learning also improves with education built around group work. Johnson and Johnson (38) analyzed student achievement in 51 high-quality studies comparing cooperative learning to so-called competitive lecture-based learning. They found that the cooperative groups on average scored 0.81 standard deviations higher (almost one grade point) compared to the lecture-based groups. Heller and Hollobaugh (28) at the University of Minnesota found that students solving complex context-rich problems in cooperative groups averaged 77 percent compared to 56 percent in the traditionally-taught section on two common final exam problems. (They could not use the complex problems for comparison because professors teaching the comparison classes felt their students could not solve such problems.)

A fourth objective concerns the research about learning and cognition. Over 1000 papers in physics have documented the inability of students to reason qualitatively about physical processes following traditional physics instruction. In courses that emphasize problem solving, lecturers often define quantities using math symbols and use these symbols to derive other principles and to solve problems. Cognitive research indicates that “the mind is ... essentially ... a symbol processing device” (39). Unfortunately, the symbols in our minds are not math symbols but are some special brain “descriptions in a sort of internal brain language” (40). A person makes sense of abstract external representations, such as acoustic signals produced by spoken language or the math symbols in an equation, by a dynamic interplay between their own internal imagery and these external representations. If the external representations have no links to a person’s internal imagery, then the person cannot construct meaning for the external representations. Written language, including the symbolic language of physics, is very abstract. For the symbols to make sense, they must elicit internal mental images that give meaning to the symbols. Technology shows promise in helping students visualize the quantities and concepts of physics (41-44) and plays an important role in the learning system described here. Familiar context in problems also helps in relating the physics with imagery in the student’s mind. (45)

A fifth objective involves a rule of thumb proposed by Arnold Arons (46). Students must see a concept or use a skill multiple times (five or six) in a variety of contexts over an extended time interval before the concept or skill becomes part of their thinking. This idea is consistent with a recent connectionist model of how the brain operates. Learning involves a network of interconnected neurons with multiple input and output pathways. At each cycle of learning, the brain wiring is altered or strengthened by some weighted response to an external stimulus. The wiring does not occur as a result of one good lecture but requires multiple interactions by the active learner (47).

In summary, we choose the following objectives and pedagogical strategies for our instructional system:

- Students develop meaningful mental images and representations for physical quantities.
- They use these images and representations to help construct the concepts of physics.
- They use the images, qualitative representations, and concepts to reason without math about physical processes.
- When the math representations of physics concepts are introduced, they are linked to the qualitative representations so that students learn to "read and write" with understanding using the math language.
- Students develop skills needed to address complex problems.
- They learn to work more effectively in groups while solving these complex problems.
- The instruction provides students with multiple exposures to conceptual and procedural knowledge over an extended time interval and in a variety of contexts.

Having identified the learning goals and the pedagogical strategies, we consider next the format for a course that has been modified to achieve these goals. We also discuss some of the curriculum materials used in the course, including examples of interactive multimedia activities.

III. COURSE FORMAT AND CURRICULUM MATERIALS

The format described here has been used with the introductory physics course for engineering students, the introductory course for science majors, and with a bridging course for poorly-prepared students wanting to become engineers (30). The course is divided into conceptual chunks—for example, Newtonian physics, work and energy, rotational motion and waves. Each chunk has a three-part format which includes:

- an overview,
- an introduction to the symbolic language in that chunk (learning to read and write with understanding using the math language of physics), and
- the application of the skills and conceptual knowledge to solve multipart problems that involving any concepts used earlier in the course.

The activities are used in large-room meetings (formerly lectures), small-room meetings (formerly labs), and in laboratories.

Qualitative Overview

In the qualitative overview for a conceptual chunk of knowledge, students help develop the concepts and learn to reason about physical processes by using qualitative representations of that knowledge. There is no math. The overview provides a math-free noise-free environment that helps students develop the visual imagery and representations on which the symbolic language can later be built.

In Newtonian physics, students use motion diagrams and force diagrams to help invent Newton's second law. This is done in the laboratory before the concept's appearance in the lecture part of the course (48). Students then use the law and the qualitative representations to answer questions that help them make sense of the world and the accepted physics principles. This often involves ActivPhysics™ multimedia simulation questions in the large-room meetings (49) and in the labs (50), and paper-and-pencil questions in the small-room meetings (51).

The type of qualitative representation that is used depends on the conceptual domains. For example, for work-energy processes, students use qualitative work-energy bar charts for their reasoning. An inverse Bungee jumping example is shown in Figure 2. After a student prediction about the initial-final energy distribution, the ActivPhysics simulation shows the initial energy distribution of the system (Fig. 2a). As the simulation runs, a second chart shows the changing energy as the process evolves. At the instant shown in Figure 2b, the initial elastic energy U_s has been converted partially into the jumper's kinetic energy K and gravitational energy U_g. If the jumper hits the support at the top of the spring, internal energy U_{in} is produced. Energy conservation is apparent.

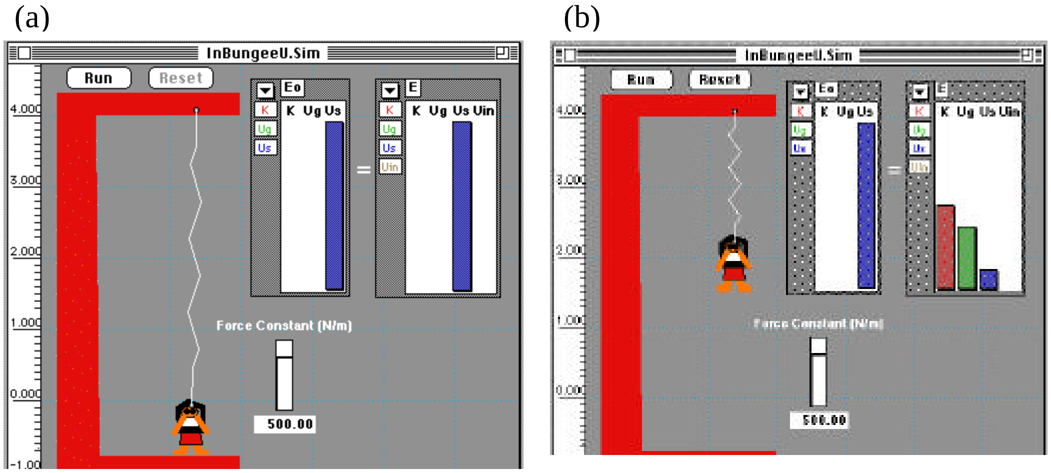


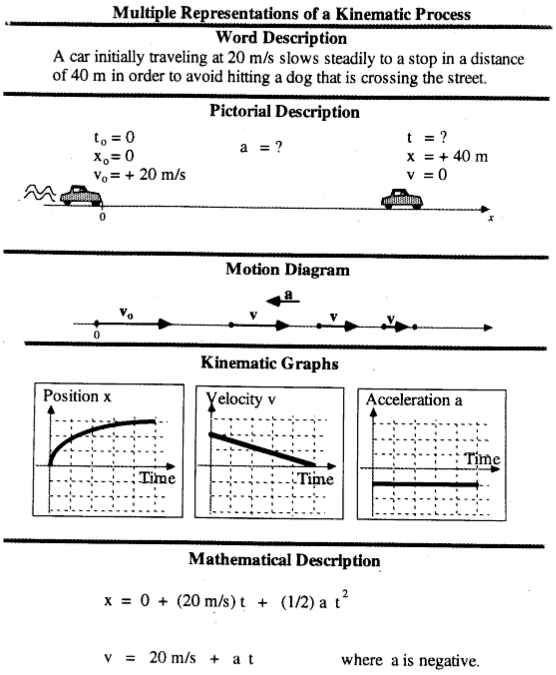
FIGURE 2. An ActivPhysics simulation that illustrates an energy transduction process using a qualitative work-energy bar chart.

Introducing the Math Representation

When young children learn to read and write, the symbols are linked to pictures that provide imagery and context for the words. The words become meaningful in part because of this imagery.

Much of traditional physics instruction is devoid of imagery. New physics principles are introduced using math derivations that involve principles developed earlier in math form. For example, a common derivation of the work-energy principle combines the math definition of work with Newton's second law and the definition of acceleration, also in math form. Research indicates that students are not very successful in answering qualitative questions about quantities such as force and acceleration. A mathematical derivation of a new concept using symbols and concepts that are poorly-understood does not lead to understanding of the new concept nor does it endear students to the study of physics.

To address this difficulty, we build the math descriptions of physics concepts on qualitative representations that students have hopefully learned to understand during the overview. Much of the activity in this part of the instruction involves the descriptions of processes in multiple ways. These descriptions provide links between the abstract math descriptions and the qualitative picture-like and diagrammatic description. A multiple description of a kinematics process is shown in Figure 3 (51).

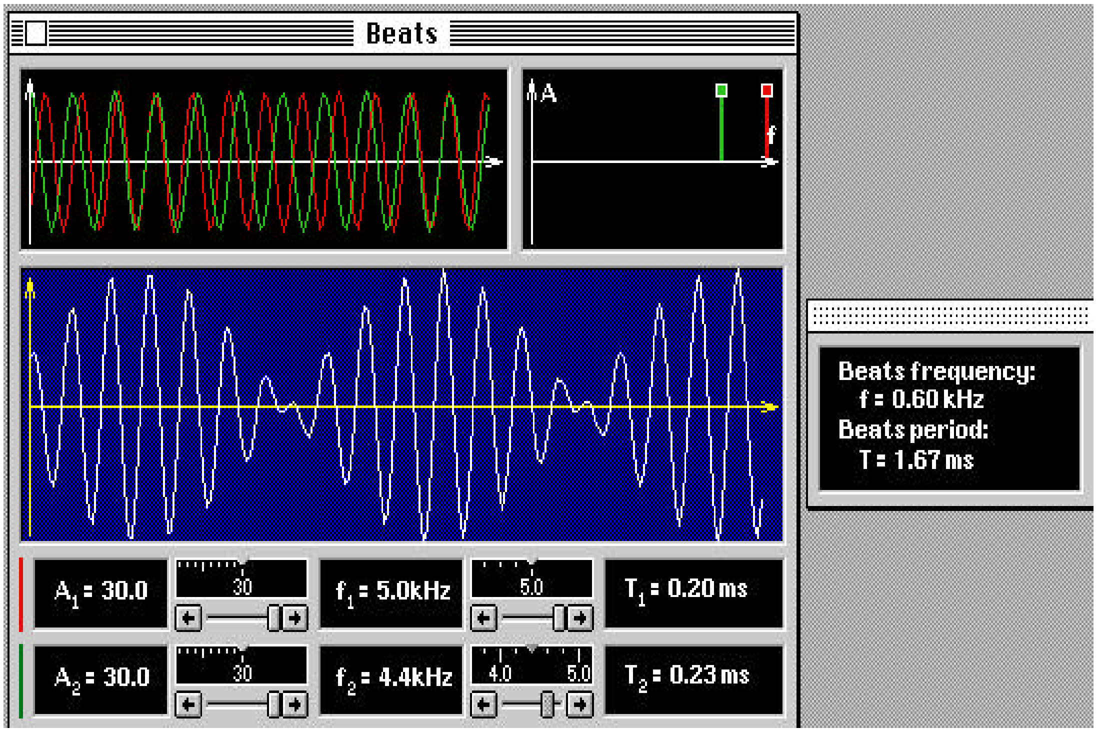


A car initially traveling at 20 m/s slows steadily to a stop in a distance of 40 m in order to avoid hitting a dog that is crossing the street.

$x = 0 + (20 \text{ m/s})\, t + (1/2)\, a\, t^2$

$v = 20 \text{ m/s} + a\, t$ where a is negative.

FIGURE 3. A Multiple Representaion of a Kinematics Process.



Beats frequency:
f = 0.60 kHz
Beats period:
T = 1.67 ms

A_1 = 30.0
f_1 = 5.0kHz
T_1 = 0.20 ms

A_2 = 30.0
f_2 = 4.4kHz
T_2 = 0.23 ms

FIGURE 4. A screen shot of an ActivPhysics beat simulation.

The ActivPhysics™ Multimedia product provides multiple representations for many processes as the processes evolve. For example, in the unit on waves, students can see a beat waveform, see the two waves producing the beats, see a frequency spectrum of the two waves producing the beats, and hear the beat sound (Figure 4).

As students start to develop understanding, they can be asked to “read” an equation and then describe a process that is consistent with the equation. Their description can involve words, pictures or some other more intuitive representation. We call these Jeopardy problems. In the example shown in Figure 5, students are to invent a process represented by the equation—there are several possibilities.

Jeopardy Problem
Math to **Bar Chart** to **Words and Sketch**

$$(100\ \text{kg})(9.8\ \text{N/kg})(50\ \text{m}\ \sin 37\ ) = (1/2)\ k\,(50\ \text{m})^2$$

FIGURE 5. A conservation of energy Jeopardy problem.

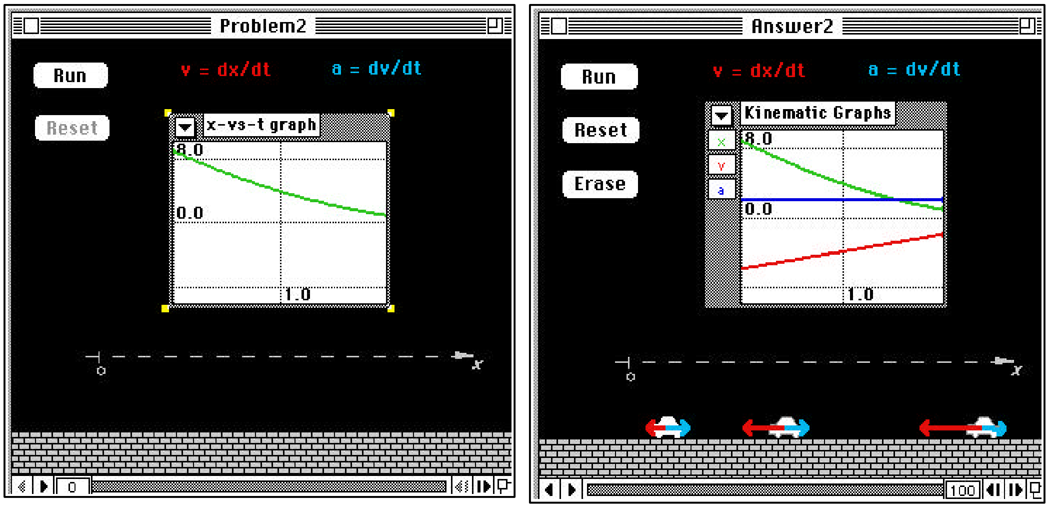
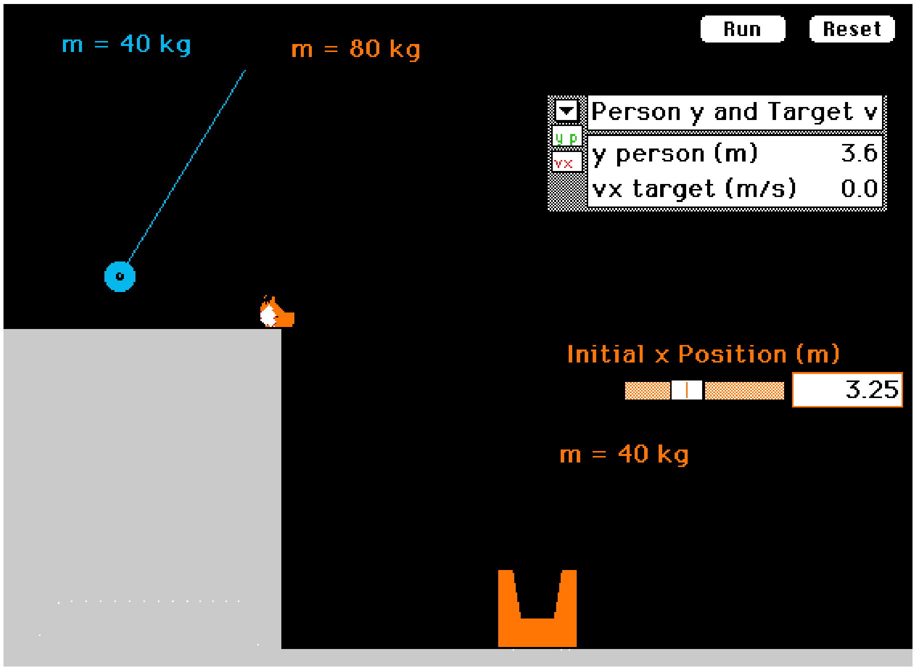

(a)
(b)

FIGURE 6. (a) An ActivPhysics kinematics Jeopardy problem and (b) its answer.

ActivPhysics also has Jeopardy problems. A kinematic position-versus-time graph produced in real time on a simulation is shown in Figure 6a. The student is asked to draw a velocity-versus-time graph, a constant acceleration-versus-time graph and a motion diagram that is consistent with the position graph. After completing their predictions, the student runs another version of the simulation which shows the motion of a car and the other graphs that are consistent with the position-versus-time graph (Figure 6b).

Complex Problem Solving

Having developed better qualitative understanding and facility with the math language used in physics, the student is now ready for regular physics problems and especially for more complex multipart problems. To solve these latter problems, students learn to add definition to poorly-defined problems, divide complex problems in parts, access the appropriate knowledge to solve each part, choose quantities whose values must be determined in order to solve the problem, make rough estimates in order to supply missing information, and justify approximations. Because the problems involve concepts from any part of the previous study, students get additional opportunities to apply previously learned concepts. An example of an ActivPhysics multipart synthesis simulation problem used in a large-room meeting is shown in Figure 7.



Person y and Target v

y person (m) 3.6

vx target (m/s) 0.0

Initial x Position (m) 3.25

FIGURE 7. An ActivPhysics synthesis problem. The medicine ball swings down and knocks the person off the ledge. Students decide where to locate the box with a padded seat so that the person lands in it and how fast the box will move after the person lands in it.

Students also solve experiment problems (52) in the labs as well as in a large-room meeting (see an example in Figure 8). Students working in more formal groups solve context-rich problems in their small-room meetings. (27-28) The engineering students also solve a 48-minute group problem that counts 20 percent of their score on exams—one grade for all persons in the group.

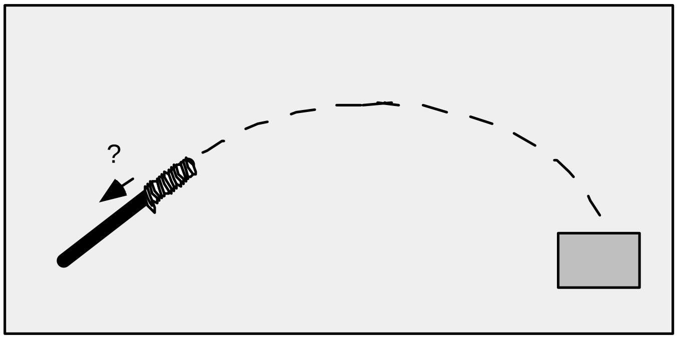


FIGURE 8. A spring-launch experiment problem. Students determine how far back to stretch the spring so that when released, it lands in the box across the room.

IV. ACTIVE PHYSICS ™

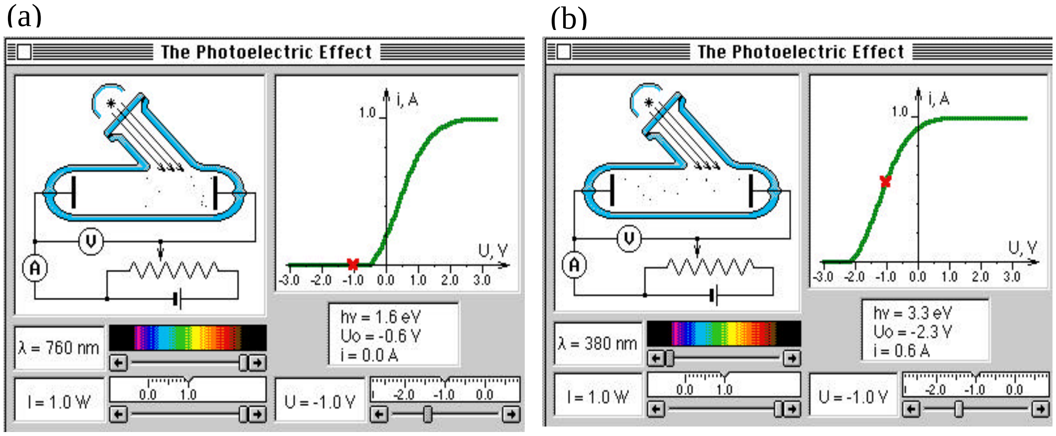
Technology shows considerable promise for improving student learning. The interactive multimedia tool used in this course is called ActivPhysics™ and is a joint project involving the author, Addison Wesley Interactive (a subdivision of the Addison Wesley Longman Publishing company—the subdivision develops stand-along interactive multimedia products), and SC Physicon (a Russian scientific and technical programming company that develops a variety of science-based models and simulations). ActivPhysics includes two CDs with activities for most conceptual areas in introductory algebra and calculus-based physics courses. A set of worksheets accompanies the CDs. The sheets can be used interactively in large-room meetings, by small student groups in labs and by students individually at home. Why should such a tool improve student learning?

Hunter brains focus on change: The physical structure of our minds evolved over many millions of years. We spent over 99.99 percent of that time as hunter-gatherers. Our minds evolved so that “Unexpected or extraordinary events ... have fast access to consciousness, while an unchanging background ... is shunted into the background.... We are streamlined to respond to the onset of an event, and then the offset” (53). Our senses and mind notice the distant flashing light of an ambulance more than brighter nearby static objects. Multimedia provides moving images and representations that have a better chance of entering our consciousness than the more static pictures or equations on the page of a book or on a blackboard.

Simultaneous representations of phenomena: When looking at static pictures, words, or equations in a book or on a blackboard, the mind focuses on them individually. It is more difficult to integrate the ideas—for example, to relate the words that describe acceleration and the description of acceleration by a graph or a motion diagram. With multimedia, we can observe the motion and the simultaneous description of the motion by a graph and a motion diagram. A quantity becomes better linked in our minds to the description of that quantity by these qualitative representations—called “perceptual enhancement” by Larkin and Simon (54).

Examining and reexamining change: Demonstrations often occur quickly. It may be difficult for our minds to observe and analyze the process or important parts of the real process. With multimedia, we can step back and forth between a time just before and a time just after some important event in a process, such as a collision. We can observe the collision and a bar-charts conversion of kinetic energy to internal energy at the instant of the collision.

Adjusting Parameters: With multimedia, it is easy to adjust parameters. For example, we can observe the photoelectric effect using long-wavelength photons with enough energy to remove an electron from a surface but too little to cause the electrons to cross a –1.0-V stopping voltage (Figure 9a). We can then reduce the photon wavelength and cause a photocurrent (Figure 9b).



(a) The Photoelectric Effect

λ = 760 nm

I = 1.0 W

U = -1.0 V

hν = 1.6 eV
Uo = -0.6 V
i = 0.0 A

(b) The Photoelectric Effect

λ = 380 nm

I = 1.0 W

U = -1.0 V

hν = 3.3 eV
Uo = -2.3 V
i = 0.6 A

FIGURE 9. (a) The 760-nm photons have enough energy to knock an electron out of the plate but the electrons do not have enough energy to cross the tube with its –1.0-V stopping voltage. In (b) the 390-nm photons cause a photo current.

We can ask "what if" questions and then actually change a parameter to see if a prediction is correct. Or we can ask students to adjust some parameter to make something happen on the first try. They can test their own prediction by trying the simulation experiment—difficult to do with real demonstrations.

Backdrop for conversation: Roth (41) suggests that much of a student's cognitive development about science concepts can occur during conversations about diagrams, graphs, and equations. Computer simulations "serve as a backdrop and referent for students' conversation" and "server to coordinate students' verbal and nonverbal communicative acts."

Easy to Use: For many professors, the preparation of classroom demonstrations is done the day before a class and the setup and take down are done in the short time interval between classes. Once a computer and projection system is installed, multimedia provides interactive demonstrations that can be used easily for every class meeting.

Do simulations represent the real world? Many professors are rightly concerned that students regard simulations as just another form of the Saturday morning cartoons. In our brief experience, this has not been a student concern. However, we do address this problem by using the simulations to model some real experiments both during the large-room meetings and more recently during the labs (50). Thus, students get experience using the multimedia simulations as a modeling tool for the real world—an important process in modern science. The other reinforcement for the authenticity of the computer simulations is that students' correct applications of the concepts of physics agree with meter readings in the simulations.

V. ACTIVE LEARNING IN LARGE-ROOM MEETINGS

How do we use these tools interactively in large-room meetings? Books on cooperative learning discuss methods for increasing student involvement. Often the professor provides a 10-minute minilecture (55). The lecturer then poses a question which students answer individually and again after discussions with neighboring students. This method allows all students to participate without baring their souls in front of a large number of other students. But the method does not provide feedback to the professor from students nor does it allow the professor to provide Socratic interactions with the students based on what they have said.

An electronic classroom communication system such as Classtalk does allow a professor to see the student responses and also allows students to see the anonymous responses of the class as a whole.

Glenn Julian of Miami University of Ohio has a low-tech version of Classtalk. The professor poses a question or problem and then after a short time interval moves up into the classroom (230 seats in the author's case). The method has been described as "coming out of the box" (56). Casual observations of student work quickly reveal times when the question or problem is unclear or when students need additional guidance. After getting used to the system, students feel free to ask clarifying questions as the professor passes. Coming out of the box also produces more bonding between the professor and students—difficult in large lecture rooms.

In addition to minilectures, Classtalk, and coming out of the box, other strategies help make large-room meetings more interactive.

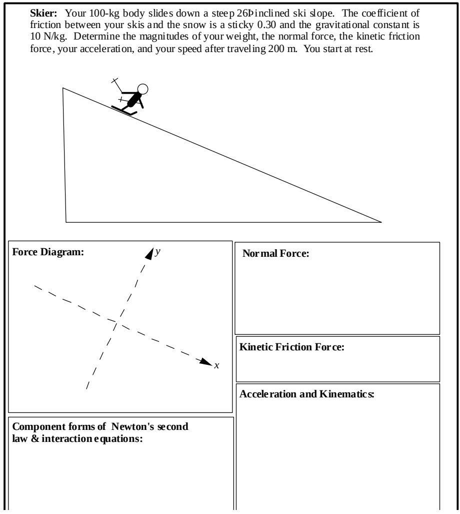
Explain method: During the first large-room meeting, provide a brief explanation of the method that you will use and the reasons for using this method. Show data that indicates that student learning improves when they become active participants in that learning. Students do not want a 30 minute lecture on pedagogy but a 5-minute discussion sets the stage. Just as students do not learn a physics concept in one exposure, you may have to remind them from time to time why you are using this method.

Student bonding during first trial: On the first class, have the students introduce themselves to students sitting on each side and in front and in back. They can share their summer experiences or some other personal information—starting to bond in preparation for their later discussions together. Following this discussion, pose the first question—like a pretest question about the first subject to be discussed. Let them know that you do not expect them to answer correctly but are simply showing them how the system works.

Worksheets: Provide worksheets on which students do their work. A sheet includes the problem or question statement and space for certain types of response—for example, constructing a motion diagram or a force diagram. The sheet may also include pictures. The sheets allow students to leave the large-room meeting with accurate records of the questions and problems, their responses, and notes about how their thinking may have changed as a result of the activities. The copying and distribution of worksheets becomes a significant task for the professor if done for each class. The system works nicely if the kit of worksheets for the semester or quarter is prepared ahead of time and distributed for student sale through a bookstore or copy center. Students bring the kit to each class.

Individual and group response: A useful method for getting student involvement involves the following routine. The question is posed. Students make individual predictions. They then discuss their responses with neighboring students and make a better group prediction. Students then get feedback as the professor runs a new version of the simulation or performs a demonstration experiment. The professor then leads a Socratic interaction to help students understand their own or other students thinking. The professor might ask students to make a new representation of the process (for example, a motion diagram to decide the acceleration direction) and to then apply some principle, such as Newton's second law, to see if their answer is consistent with the diagram and the principle. Students learn much more by doing their own reasoning to arrive at new thinking rather than having the professor do it for them.

Breaking problem into small cognitive tasks: For interactive simulation problems during the beginning of a conceptual area, the problem can be broken into several small tasks—for example, constructing a free-body diagram, using the diagram to apply Newton's second law in component form, determining the magnitudes of the normal and friction forces, determining the acceleration, and determining the speed of an object after traveling a certain distance (see Figure 10). After each step, you can run a version of the simulation which allows students to check their result. You can sometimes inject into these problems qualitative questions that challenge alternative conceptions.



Skier: Your 100-kg body slides down a steep 26° inclined ski slope. The coefficient of friction between your skis and the snow is a sticky 0.30 and the gravitational constant is 10 N/kg. Determine the magnitudes of your weight, the normal force, the kinetic friction force, your acceleration, and your speed after traveling 200 m. You start at rest.

Force Diagram:

Normal Force:

Kinetic Friction Force:

Acceleration and Kinematics:

Component forms of Newton's second law & interaction equations:

FIGURE 10. A multiple representation worksheet for a dynamics problem.

Planning a solution: As their expertise increases, a problem solution starts with student groups planning a strategy. Individuals need quiet time to make their own individual plan before discussing it with neighboring students. Planning is especially important for the more complex synthesis problems, experiment problems, and context-rich problems. Students break the big problem into several small problems and decide the best conceptual knowledge for solving each small problem and the quantity they need to determine for each part. For large-room meeting synthesis problems, the author prefers a brief discussion following the planning to be sure that most of the informal groups are moving in the right direction. During small-room meeting group problem solving, the groups can be left more on their own. The professor or teaching assistant moves from group to group to see how students are doing.

Patience: Perhaps the most difficult part of a system such as this occurs in the first ten seconds after posing a question or asking them to solve a problem or part of a problem. The classroom may seem very quiet and students may be looking at the professor with what seem like blank stares. The ten seconds seems like an eternity. The students are thinking but need time to reflect on the questions and pull their thoughts together. The professor must wait a long time (30 seconds or a minute) before the room starts buzzing. It will be difficult at first, but be patient. Be sure the question is stated clearly.

Reduced content: One final concern is the almost certain omission of traditional parts of the curriculum if the students do much of the thinking and talking in the large-room meetings. If one looks carefully at the thousands of papers that document the lack of learning during traditional instruction, it becomes clear that rushing through the curriculum in a didactic mode is very close to the identity educational transformation operator shown in Eq.(2). The students learn little about a lot. The reduction in content in a system such as described here does not concern our former physics majors who are now out in the workplace. They say that it is more important to learn to learn (the SCANS Report) than it is to acquire lots of conceptual knowledge (see Fig. 1). Instead of showing concern about reducing the content and letting students do the work, we should show concern for a didactic system in which students learn almost nothing about a lot.

V. SUMMARY

Interactive multimedia offers advantages in helping students form useful qualitative representations for physical quantities and concepts. These qualitative representations are used to help develop meaning for math descriptions of physical process. Students represent physical processes using multiple representation techniques. As students develop understanding for the meaning of the symbols, they learn to "read an equation" by solving Jeopardy problems which start with an equation or some other representation of a process and invent a physical process that is consistent with the equation or other representation. Finally, students solve more complex multipart experiment problems, multimedia synthesis problems, and context-rich problems. Much of the work is done by informal student groups during large-room meetings and more formal groups during the small-room meetings and labs.

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